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MODELING AND OPTIMIZATION OF THE COMBINED TRANSPORTATION SYSTEM

MODELOWANIE I OPTYMALIZACJA KOMBINOWANEGO SYSTEMU TRANSPORTOWEGO

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Abstract: *Transport is one of the most important logistical subsystem. It occurs at every stage of the logistic chain. Efficiency of transport utilization is an important element determining the financial results of an enterprise. In addition to the use of transport systems, organizational and management issues also play an important role in this matter. Traditional approach sometimes seems to be insufficient, especially in the case of complicated transportation tasks. The solution in this case is the use of modeling and optimization of the transport system used by the company, that has been presented in the paper, on the basis of the selected example. Optimal use of transport provides tangible benefits and allows to achieve competitive advantage in the market.*

Streszczenie: *Transport jest jednym z najważniejszych podsystemów logistycznych. Występuje on na każdym etapie funkcjonowania łańcucha logistycznego. Efektywność wykorzystania transportu jest istotnym elementem decydującym o wynikach finansowych przedsiębiorstwa. Oprócz wykorzystywania systemów transportowych istotną rolę odgrywa tu również kwestia organizacji i zarządzania. tradycyjne podejście czasem wydaje się niewystarczające szczególnie w przypadku wystąpienia skomplikowanych zadań transportowych. Rozwiązaniem w tym przypadku jest wykorzystanie modelowania i optymalizacji wykorzystywanych przez przedsiębiorstwo systemów transportowych co też przedstawiono w referacie na podstawie wybranego przykładu. Optymalne wykorzystanie transportu zapewnia uzyskanie wymiernych korzyści i pozwala na uzyskanie przewagi konkurencyjnej na rynku.*

Key words: *transport system, transport systems modeling, transport systems optimization*
Słowa kluczowe*: system transportowy, modelowanie systemów transportowych, optymalizacja systemów transportowych*

INTRODUCTION

The theme of this paper is the issue of modeling and optimization of combined transport system. Logistics has become one of the main areas for gaining competitive advantage. Transport occurs at every stage of the logistic chain. Therefore, choosing the right transport system is a key challenge for companies seeking to achieve competitive advantage. The subject of the paper is modeling and optimization of the transport system used in the chosen company.

The purpose of the paper is to analyze modeling and selected methods of transport optimization in the enterprise, user of combined transport, and to choose a less complex, efficient method which allows to optimize the transport process using generally available analytical tools. The inspiration for choosing the theme of the paper is the increasing role of the use of combined transport in modern companies, which at the same time are struggling with the problem of its efficient use. In order to achieve the intended effect, the theory of modeling and optimization of transport systems was used to analyze selected elements. The paper also presents selected aspects of modeling and optimization of transport systems. In addition, based on data obtained from the company, the model was built and the combined transport system used by the company was optimized using the selected method and analytical tools. It has been assumed that by using less sophisticated methods and generally available analytical tools, we are able to optimize transport system within the enterprise.

1. MODELING OF TRANSPORT SYSTEMS

Concept of model and modeling

Modeling is defined as a sequence of steps that results in a specific model. The concept of a model is understood as a thinkable or material realization of a system that reflects or reproduces the object of action, which is capable of replacing it so that its investigation provides us with new information about the object (Brzeziński M., 2007).

Considering this, we can assume that the model is a mapping of reality or its fragment. The system model is a quantitative and qualitative representation of the system under consideration on a different material basis, capturing those characteristics of the system that are relevant from the point of view of the conducted research. This means that the model is treated as a simplified representation of reality, i.e. its essential characteristics, relevant for the intended purpose of the study (Jacyna M., 2009).

When designing a model, it is necessary to take certain assumptions so that the model can fulfill its role. These assumptions are as follows (Brzeziński M., 2007):

• awareness of the purpose of the model;

• reflect the elements, their properties and the relationship between these elements and properties;

• internal conformance of the model and the information underlying its design;

• include the relationship between the real fragment of reality and its environment in the model presentation.

Infinitely many variables affect the object – it is impossible to build a model that faithfully reproduce behavior of an object with all possible external influences. In the model, we limit the number of "connections" with the environment to these, which influence on the behavior of the object is important for the analysis (Jacyna M., 2009).

The model takes into account only selected, affecting factors and only within a limited range of variability. The scope of the phenomena which are taken into consideration depends on the available knowledge and the purpose of the simulation research. Models of complex systems are built hierarchically, describing system components. From simple models, taking into account the interaction between elements, a complex model of the system is built.

Several types of models are used in the process of system description (Jacyna M., 2009):

• linguistic (verbal description);

• graphical (e.g. circuit diagram, diagrams of characteristic);

• mathematical.

The ultimate goal is a mathematical model. It can be obtained through the use of fundamental laws / principles of technique to describe the system (physical modeling) or by mathematical record of system functions without physical analysis (functional modeling).

The classification of mathematical models, that take into account the passage of time, the uncertainty of the properties of the projections of the characteristics of the structural elements and the form of model’s equations of the system and environment, is shown in Figure 1.

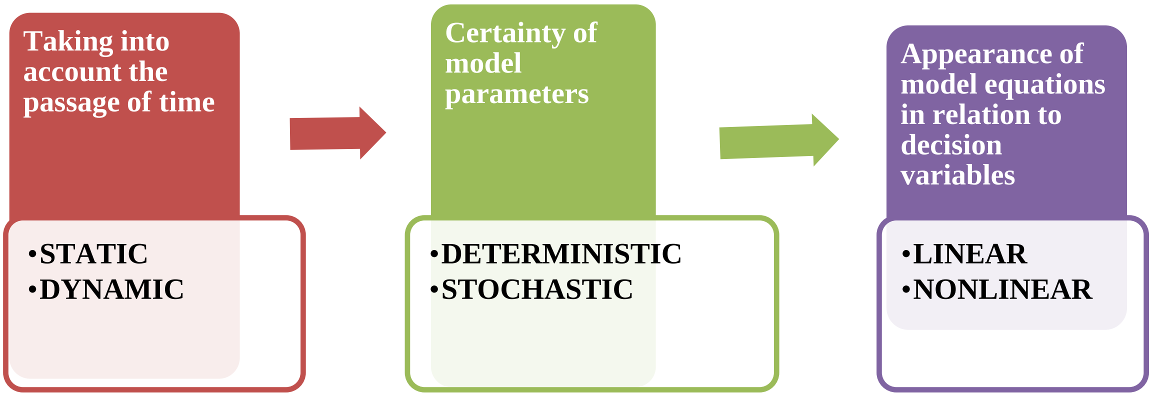


Fig. 1. Simplified classification scheme of mathematical models

Source: Own elaboration based on Jacyna M., *Modelowanie i ocena systemów transportowych,* Oficyna Wydawnicza Politechniki Warszawskiej, Warszawa 2009

The essence and stages of the transport system modeling process

An important aspect of modeling is the definition and characterization of the modeling process of the transport system. The process is understood as a sequence of actions that follow each other in an orderly manner, with clearly identified causes of their occurrence, and are subsequent stages of development. The definition of a process is closely related to the definition of the system in which it is implemented. It can also be stated that processes are the essence of the functioning of every system (Jacyna M., 2001).

The modeling process will, therefore, be a sequence of actions (operations) designed in such a way that the main objective of the research is realized. It is important for the process to have the logical and simplest possible course, then the best efficiency in achieving the goal will be obtainable.

The system modeling algorithm is shown in Figure 2.

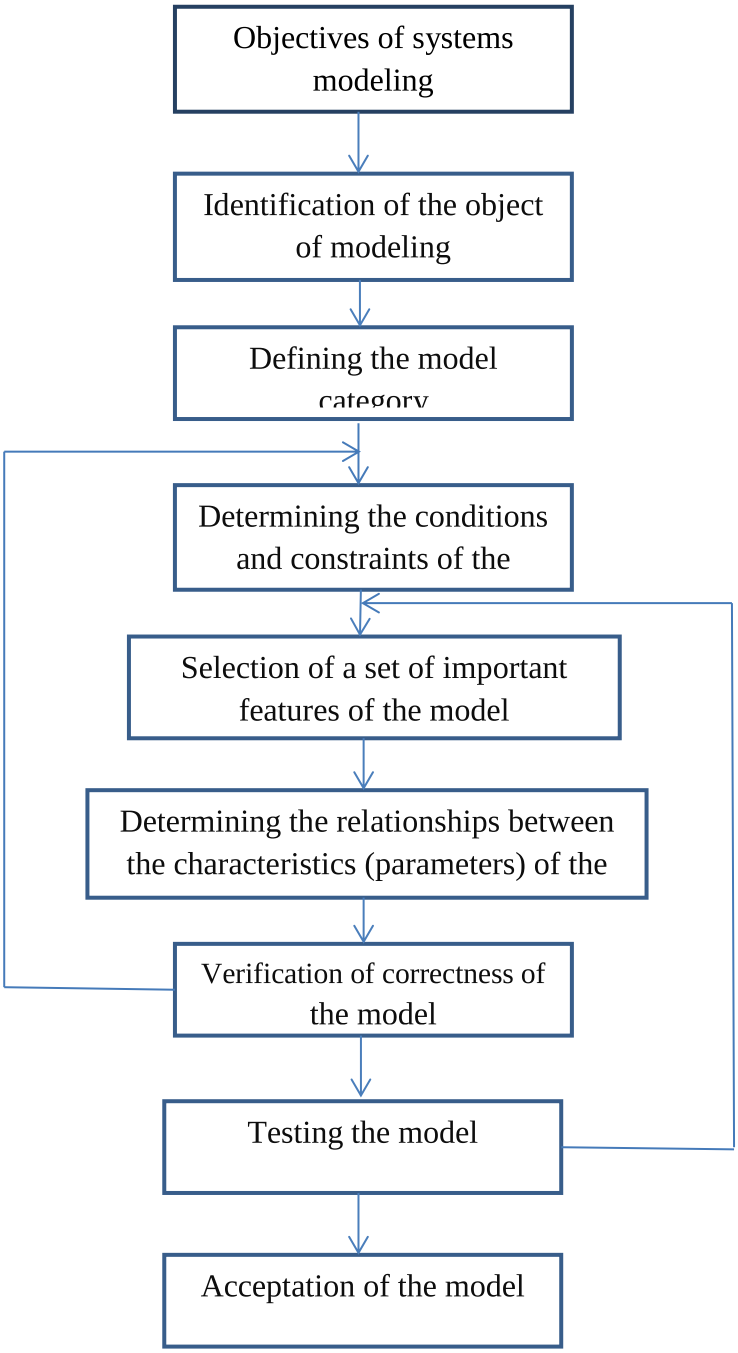


Fig. 2. Algorithm of the modeling process

Source: Own elaboration

Modeling of the transport system is aimed at understanding the dependencies of the actual transport object on the basis of a properly selected model. Model and experiments carried out with the model allow to explore mentioned dependencies and achieve the goal of research. These goals can be cognitive, like e.g.:

• identification of a system;

• designation of the relationships between the quantities occurring in the system under research;

• designating the course of variability of the quantities that may be tested during the research.

The subject of research may also be utilitarian purposes, including: (Jacyna M., 2009):

• searching for optimal solutions, e.g. distribution of traffic flow;

• analysis and evaluation of variants of the technical and organizational equipment of the transport system;

• planning changes to the technical and organizational equipment of the transport system;

• scheduling the transport process during the realization of specific transport needs having fixed technical and organizational equipment of the transport system;

• control of the course of the transport process, given its initial state, specific inputs, and criteria for evaluating the quality of the process.

Before writing the decision-making problem in mathematical form, it is necessary to specify:

• parameters - known or defined values, unchanged when solving a given problem;

• decision variables - sought values that need to be designated when solving a problem;

• constraints - algebraically expressed by the system of equations and inequalities with respect to decision variables;

• criterion function - solution quality indicator, algebraically related to decision variables.

The first, essential step in the process of modeling transport systems is to identify the system itself and the system environment.

The second major step in the process under consideration is the use of methods and tools of assessment identified for the system.

Next, important step in the process of modeling a transport system is formation of a specific model of a system process based on any of the existing methods.

Elements of the transport system model will be assumptions and limitations of the objective function. It is acceptable to adopt certain criteria that determine the system, properties of these elements will be the values of the given criteria – i.e. various types of numerical values, percentages, indicators and measures, both maximized and minimized.

The subsequent stage of the process will be implementation of the received model of transport system. The phenomenon of the implementation of the model, and therefore its incorporation into reality, will transform almost completely the functioning of the logistic system of the logistics services provider. This hypothesis leads to the next stage of the transport system modeling process, control and evaluation of the implemented model.

The above stage is the final phase of the transport system modeling process. It is essential for the proper course of the process, as it confirms or negates the correctness of the process. Constructing the mathematical model of the system under investigation or the relations occurring in the system allows future formulation and solution of various optimization tasks.

3. OPTIMIZATION OF TRANSPORT SYSTEMS

Optimization is the designation, in accordance with a defined course of conduct, of the best solutions from the viewpoint of the accepted criterion, appropriate under the given conditions and meeting the constraints imposed, and therefore coming from the set of permissible solutions. In the general mathematical sense, optimization is the procedure leading to the definition of an extreme solution, accordingly to the criterion expressed in a form of minimum or maximum on the set of X values.

The practical purpose of optimization is to bring the decision maker to make an optimal decision, i.e. the best from the viewpoint of decision-making criterion, that mathematicians call the function of the objective or function of the criterion. The most commonly used optimization criteria in the area of transport systems will be (Rydzkowski W., 2010):

• criterion of the minimal cost of the transport process, within a given time limit;

• criterion of minimal time of the transport process, under given conditions, including cost limitation

In one-criterion optimization, all the information needed to design a solution should be collected before the calculation begins, while the decision-making process is often a multi-step in nature and conducted studies influence the solution. Often, there is a need to add a new criterion during the research (Jacyna M., 2009).

In some cases, significant factors cannot be reliably taken into account, or absolute optimization leads to violation of constraints difficult to formalize or model (for example, ethical or political constraints). The aim is then to satisfy the aspirations at a certain level. The enriched formalism of multi-criterion optimization better reflects complex decision problems.

In other words, multi-criterion optimization is the most real, closest to human and the most natural concept of decision-making process. It occurs wherever the resulting value depends on many factors (criteria, dependencies, constituents, constraints), when a number of objective functions have to be included in the decision-making task, and consists in finding the best compromise combining the "interests" of those functions.

The optimization problem is formulated as a search for the values of N variables *x1 ... xN* so that the value of the function *F(x1 ... xN)* of these variables is extreme. Variables *x1 ... xN* are called the decision variables, and the function *F(x1 ... xN)* is defined as the objective function that is maximized or minimized (Rydzkowski W., 2010). The optimization problem can thus be described as follows:

$$F(x_1 \ldots x_N) \longrightarrow max \quad \text{or} \quad F(x_1 \ldots x_N) \longrightarrow min \ldots\ldots(1)$$

where:

$x_1 \ldots x_N$ - decision variables

N – number of variables

The solution of optimization tasks using the developed mathematical model of the system or the selected relationship may be sought in a continuous or discreet manner. The approach depends on both, the way mathematical model is formulated and the available input data to the model. In case of the search for a solution in a continuous manner, the model and input data should allow for the determination of the objective function F(x1 ... xN) value at any point in the criterion space. Standard optimization methods are used for determining the extremes of a multivariable function. From the practical point of view of conducting systems research, continuous approach is sometimes unfeasible. On the other hand, a discrete approach to the optimization tasks requires a multi-criterion evaluation of the solution options (Jacyna M., 2001).

Table 1 summarizes selected system optimization methods and presents their advantages and disadvantages

Table 1. Advantages and disadvantages of applied methods of evaluation of logistic systems

Method of evaluation and optimization of systems	**Advantages**	**Disadvantages**

Quality assessment method	✓ Simple method without need for accounting calculation ✓ Fast evaluation of suppliers	o Low suitability in the case of a large number of criteria and / or suppliers o Superficial assessment o High level of subjectivity in evaluation
Arithmetic mean method	✓ Simple method, simple accounting calculations ✓ Point system in the evaluation of the suppliers	o In case of the same evaluation of several systems, there are difficulties with an unambiguous choice o Suitability for a small number of systems and criteria, with clear differences in evaluations of individual criteria o Low level of objectivity in the allocation of ratings for criteria considered o Simplified scoring scale for a given criterion
Weighted average method	✓ Low difficulty of the method ✓ Differentiate the criteria by giving them the appropriate weight	o As in the previous method, in the case of an equal assessment of several systems, there is a problem with choosing the optimal solution o Usability mainly in the case of a small number of systems and criteria o Subjectivity in assigning weight to a criterion
Linear programming	✓ Ability to use a widely available tool - Solver (easy to use) ✓ High popularity of the use of the method ✓ Ability to introduce constraints, decision variables ✓ Linear computational complexity of the algorithm	o The main drawback of linear programming is that not all can be expressed using numbers.
Nonlinear programming	✓ Universal availability of computing packages ✓ Ability to use effective algorithms to determine optimal solutions and approximate solutions of multiple classes of nonlinear programming problems ✓ High accuracy of the method	o It is difficult to obtain an analytical solution o Required knowledge of derivative functions and differential conditions

AHP Method	✓ Synthetic indicators for evaluating suppliers ✓ Comparison of the criterion with the other ✓ Comparison of variants with respect to the criteria under consideration ✓ The method is quite easy In use ✓ Great popularity among decision makers	○ Usage of arbitrary scale of ratings. Values are determined by verbal definition made by the decision maker, when the meanings of verbal terms differ depending on the subject matter and on the set of elements on which the comparison is made. ○ High sensitivity of the method to minor changes, e.g. the possibility of ranking variants in the case of equivalence of all ○ Change of ranking when adding a criterion for which all variants are equivalent (impact on aggregate weights of variants)
ELECTRE III Method	✓ The expectations and preferences of the decision maker are modeled in a flexible manner ✓ Introduction of overlapping relationship, which allows to organize variants ✓ Assumption of incomparability of criteria, hence the abolition of their aggregation	○ Not applicable in a direct way in the hierarchical approach because of the possibility of obtaining (in the particular case) order without preference for selected alternatives . ○ Possibility of incomparability of variants when the decision maker determines too low veto threshold
Graph method	✓ Give relative importance (weight) to each criterion ✓ Use of additional indicators (compliance indicator, non-compliance indicator) ✓ Ability to graphically interpret solutions	○ Rather labor-intensive method ○ Lack of tools (e.g. Computer) supporting the use of the method

Source: Own elaboration

4. MODELING AND OPTIMIZATION OF THE TRANSPORT SYSTEM OF THE SELECTED ENTERPRISE

The presented aspects of the modeling process of transport systems are the basis for the further part of the paper, in which the model of the transport system will be built on the basis of data from a actual company operating on the market. Due to the availability of tools and the small degree of complexity of the method, it was decided to define the transport system as linear task and to use Excel and spreadsheet software.

Combined transport system model was formulated in the form of a linear programming problem and solved basing on Excel software. For solving logistical problems, from a wide range of expensive specialized computer programs, spreadsheets with built-in statistical tools can be used equally successful. One of such devices is the Solver module, which uses simplex method for linear and integer problems with constraints for variables and branch-and-bound methods, includes the following options: (Walkenbach J., 1999):

• identifies more than one of the changing decision-making cells;

• narrows the problem by additional assumptions about how input cell changes;

• generates a solution of the minimum and maximum of the specified objective function;

• can find many solutions to the optimization task.

The company producing nitrogen fertilizers, hereinafter referred to as FIRM, is not only the producer of fertilizers, but also an importer. COMPANY headquarters is located in Warsaw, while one of its main branches is located in Radom. Sales are carried out at the production facilities in Warsaw and Radom, as well as in wholesalers located in Wloclawek, Puławy, Kedzierzyn-Koźle and Tarnow. COMPANY employs native drivers to work in Poland using their own fleet. Another batch of imported fertilizer has just arrived. In Swinoujscie and Medyka there are respectively 900 and 400 tons of nitrogen fertilizer to receive. The entire fertilizer should be transported by car transportation from the border points to Szczecin and Przemysl, then by rail to the main branches in Warsaw and Radom, and, finally, from there by car transportation to wholesalers in the country. The network problem capture is illustrated in Figure 3.

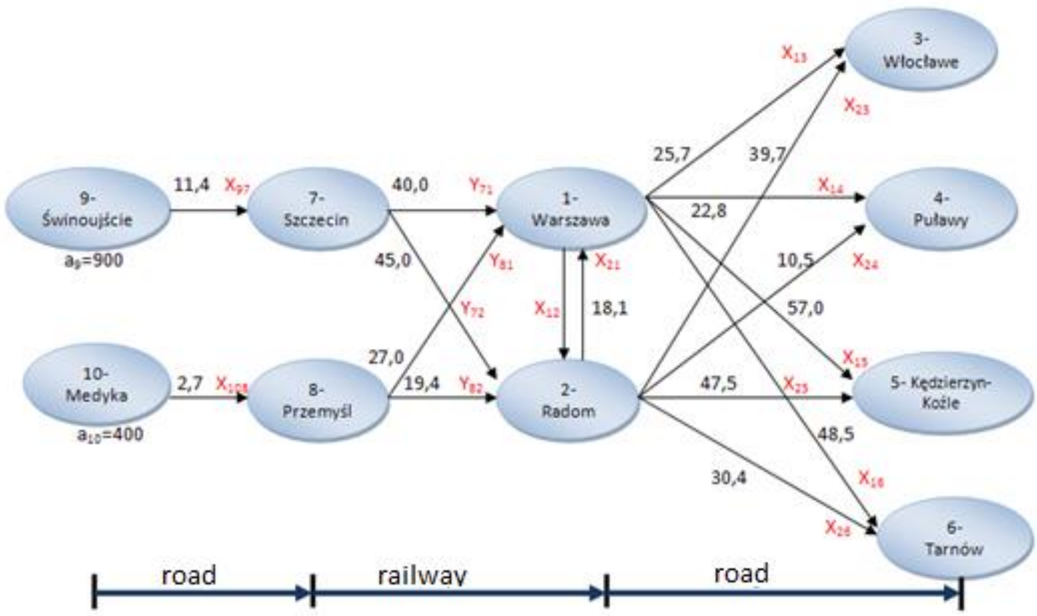


Figure 3. A network of transport related to the delivery of imported fertilizers
Source: Own elaboration

The cost of transporting one ton of fertilizers from Warsaw and Radom to wholesalers and border points to Szczecin and Przemysl are the same. The unit cost of transport is fixed at 0.19 [PLN / km]. The cost of transporting one ton of fertilizers by rail is 0.9 [PLN / km]. All transportation costs are shown in Figure 3 next to the arrows connecting the nodes. Table 2 lists the fertilizer requirements in each locality (denoted by b_j), and the upper limits for the amount of fertilizer that can currently be taken by the each wholesaler (denoted by g_j).

Table 2. Demand and enrollment capacity of fertilizers in different localities

LOCALITY	**DEMAND (b_j)**	**ENROLLMENT CAPACITY (g_j)**
Radom	300	350
Włocławek	150	200
Puławy	200	300
Kędzierzyn-Koźle	150	300
Tarnów	200	200

Source: Own elaboration

When planning the carriage, not only the condition of minimizing the cost of transport should be taken into consideration, but also the necessity of transporting by rail with complete railway draft of cars. One takes 100 tons.

At present, COMPANY is struggling with the problem of selecting the right transport method to perform the task so that the service is the most effective. Satisfying the growing demand for fertilizers in the spring and the ever increasing number of orders from customers addressed to the COMPANY, forces managers to make changes and shorten the entire transport process.

Model of a combined transport system in the enterprise

In the chosen model of combined transport, the main part of the carriage is made by rail, while the transfer between border points and the terminal, the main facilities and wholesalers are carried out by car. One of the logistic subsystems defined in such manner is a prime example of a system with periodic tasks or a multiphase system.

The first phase includes the carriage of an integrated load unit from the border points to the terminals established near these points. Phase two is the main part of carriage, carried out by rail. It is a transport of the integrated unit between terminals and branches located in Warsaw and Radom. The third phase is about delivering the transported unit in appropriate quantities from said branches to the wholesalers.

In the design of the model, the decision variables, x_{ij} expressing the flow on the arrows (i, j) and y_{pi} expressing the number of railway drafts of cars, are used. Because the fertilizers from the terminals in Szczecin and Przemysl must be exported with complete railway drafts of cars, the variable y_{pi} will be interpreted as the number of railway draft of cars used for carriage on the route (p, i). One composition takes 100 tons of fertilizer, and therefore global cost of rail transport will be:

$$K_1= 4000y_{71}+4500y_{72}+2700y_{81}+1940y_{82} \quad (2)$$

In addition to the above costs, the cost of K_2 related to automotive carriage, i.e. the sum of the unit cost on individual arrows corresponding to road transport by the adequate variables x_{ij}, should be added. It is necessary to solve the following linear optimization task:

$$K_1+K_2 \qquad \min \quad (3)$$

$$K_1= \sum_{(p,i)\epsilon Q} d_{pi}y_{pi} \quad (4)$$

$$K_2=\sum_{(i,j)\epsilon Q} d_{ij}x_{ij} \quad (5)$$

with the conditions:

for node no. 1 : $x_{12}+ x_{13}+ x_{14}+ x_{15}+ x_{16}- x_{21}-100y_{71}-100y_{81}=0$,

for node no. 2 : $300\leq 100y_{72}+100y_{82}+ x_{12}-x_{21}- x_{23}-x_{24}-x_{25}- x_{26}\leq 350$,

for node no. 3 : $150\leq x_{13}+x_{23}\leq 200$,

for node no. 4 : $200\leq x_{14}+x_{24}\leq 300$

for node no. 5 : $150\leq x_{15}+x_{25}\leq 300$,

for node no. 6 : $x_{16}+ x_{26}=200$,

for node no. 7 : $100y_{71}+100y_{72}-x_{97}=0$,

for node no. 8 : $100y_{81}+100y_{82}-x_{108}=0$, for node no. 9 : X_{97}=900,

for node no. 10: X_{108}=400,

…………………………….

$x_{ij} \geq 0$; $y_{pi} \geq 0$; (i=1,2; j=3,4,5,6,9,10; p=7,8)

y_{pi}- integer numbers (p= 7,8; i= 1,2)

Application of the simplex method for solving the optimization task

The simplex method is considered to be one of the basic methods of finding the optimum solutions of linear programming tasks. This is a method using publicly available tools. It consists of a sequential, strictly defined base solution overview. To apply it, it is necessary first to transform a set of constraint conditions into a system of linear equations. The full review of a set of base solutions is ineffective given the number of solutions and the size of each set of equations to be transformed (Sikora W., 2008).

The task of linear programming with any number of variables can be solved by setting all of the apex points of a polyhedron, and then comparing the value of the function in-point. Due to the multiplicity of points, the problem is to determine the value of the objective function and to find the optimal vertex that satisfies the task condition in Poland. The essence of the simplex method is that if any point and value in-point of the target function is known, then all vertex points in which the deliberate function assumes inferior values are discarded. The next iteration step is that we move to the next vertex, located on one edge with the already found point where the targeted function achieves better values. Iteration ends when the viewed vertex point is best in terms of the appropriate value of target function. Therefore, we omit unacceptable base solutions and values that are worse than the one currently being considered.

In the simplex method a system of linear equations is considered:

$$B \times X_B + A_N \times X_N = b$$

- in the first iteration

- and Equivalent Equations of Limiting Conditions:

$$X_B + B^{-1} \times A_N \times X_N = B^{-1}b \qquad (7)$$

in subsequent iterations of the simplex algorithm where:

$$A \times X = B,\ A=[B, A_N] = [a_{ij}]_{mxn},\ X = [X_B, X_N] = [x_1, x_2, \ldots, x_n],\ b=[b_1, b_2, \ldots, b_m]$$

The general algorithm for the simplex method is shown in Figure 4:

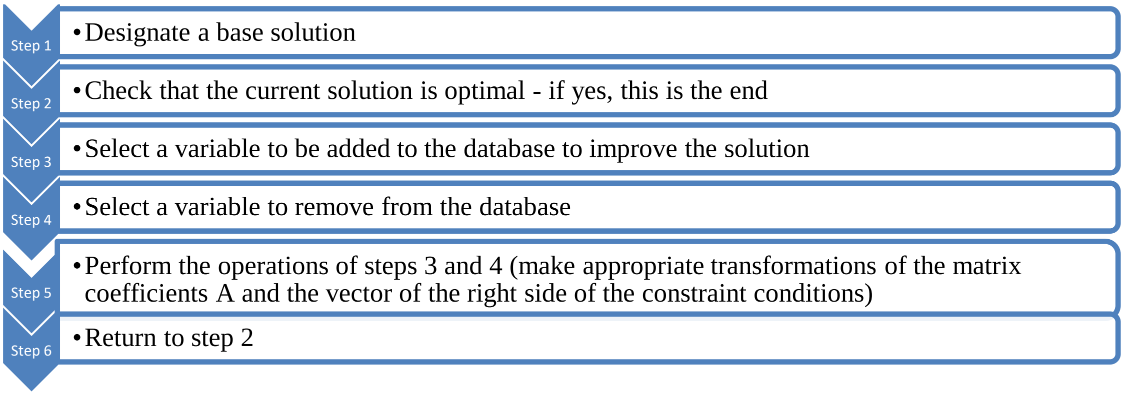
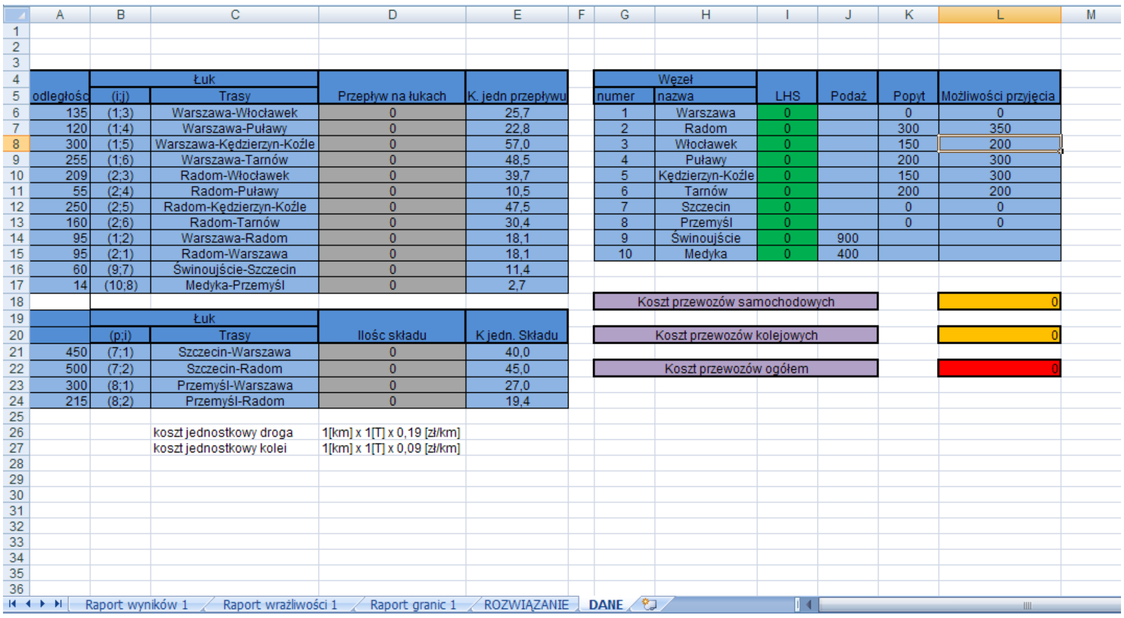


Fig. 4. Algorithm of the simplex metod

Source: Own elaboration based on Trzaskalski T., Wprowadzenie do badań operacyjnych z komputerem, Polskie Wydawnictwo Ekonomiczne, Warszawa 2008

5. IMPLEMENTATION OF THE METHOD IN EXCEL

Figure 5 shows a data sheet. Cells shaded in gray are values of decision variables, the target function is a red cell, and cells containing functions with constraint conditions are green.



odległość	Łuk (i;j)	Łuk Trasy	Przepływ na łukach	K. jedn przepływu
135	(1;3)	Warszawa-Włocławek	0	25,7
120	(1;4)	Warszawa-Puławy	0	22,8
300	(1;5)	Warszawa-Kędzierzyn-Koźle	0	57,0
255	(1;6)	Warszawa-Tarnów	0	48,5
209	(2;3)	Radom-Włocławek	0	39,7
55	(2;4)	Radom-Puławy	0	10,5
250	(2;5)	Radom-Kędzierzyn-Koźle	0	47,5
160	(2;6)	Radom-Tarnów	0	30,4
95	(1;2)	Warszawa-Radom	0	18,1
95	(2;1)	Radom-Warszawa	0	18,1
60	(9;7)	Świnoujście-Szczecin	0	11,4
14	(10;8)	Medyka-Przemyśl	0	2,7

	Łuk (p;i)	Łuk Trasy	Ilośc składu	K jedn. Składu
450	(7;1)	Szczecin-Warszawa	0	40,0
500	(7;2)	Szczecin-Radom	0	45,0
300	(8;1)	Przemyśl-Warszawa	0	27,0
215	(8;2)	Przemyśl-Radom	0	19,4

koszt jednostkowy droga	1[km] x 1[T] x 0,19 [zł/km]
koszt jednostkowy kolei	1[km] x 1[T] x 0,09 [zł/km]

Węzeł numer	Węzeł nazwa	LHS	Podaż	Popyt	Możliwości przyjęcia
1	Warszawa	0		0	0
2	Radom	0		300	350
3	Włocławek	0		150	200
4	Puławy	0		200	300
5	Kędzierzyn-Koźle	0		150	300
6	Tarnów	0		200	200
7	Szczecin	0		0	0
8	Przemyśl	0		0	0
9	Świnoujście	0	900		
10	Medyka	0	400		

Koszt przewozów samochodowych	0
Koszt przewozów kolejowych	0
Koszt przewozów ogółem	0

Fig. 5. Spreadsheet with data for determining the transport plan for import fertilizers
Source: Own elaboration

In the green, yellow and red cells are written formulas, the full list presented in Table 3

Table 3. List of formulas for the problem of transportation of fertilizers.

Name of cell	Formula	Element in model
I6	=(D14+D6+D7+D8+D9-D15-100*D21-100*D23)	Function 18
I7	=(100*D22+100*D24+D14-D15-D10-D11-D12-D13)	Function 19
I8	=(D6+D10)	Function 20
I9	=(D7+D11)	Function 21
I10	=(D8+D12)	Function 22
I11	=(D9+D13)	Function 23
I12	=(100*D21+100*D22-D16)	Function 24
I13	=(100*D23+100*D24-D17)	Function 25
I14	=D16	Function 26
Name of cell	Formula	Element in model
I15	=D17	Function 27
L18	= SUMPRODUCT (D6:D17;E6:E17)	K_2
L20	= SUMPRODUCT (D21:D24;E21:E24)	K_1
L22	=(L18+L20)	K_1+K_2

Source: Own elaboration

The figure no. 6 shows the filled Solver-Parameters window.

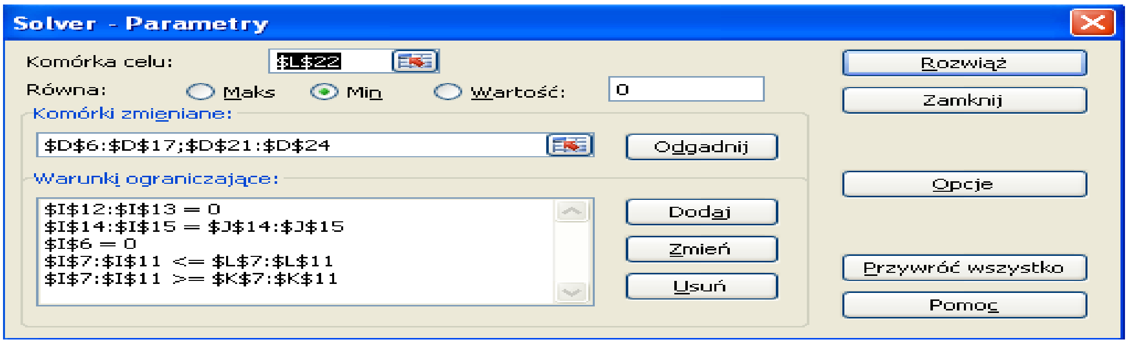


Fig. 6. Filled Solver Parameters dialog (example of transporting fertilizers)
Source: Own elaboration

The sheet with the optimal solution is shown in Figure 7. According to this solution, Warsaw accepts two railway drafts of cars from Szczecin, and will transport by car 200 tons to Włocławek. Radom will accept 11 railway drafts of cars (including 7 from Szczecin and 4 from

Przemysl) and will transport by car 300 tons to Puławy, 250 tons to Kedzierzyn-Koźle and 200 tons to Tarnow.

odległość	Łuk (i;j)	Łuk Trasy	Przepływ na łukach	K. jedn przepływu
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120	(1;4)	Warszawa-Puławy	0	22,8
300	(1;5)	Warszawa-Kędzierzyn-Koźle	0	57,0
255	(1;6)	Warszawa-Tarnów	0	48,5
209	(2;3)	Radom-Włocławek	0	39,7
55	(2;4)	Radom-Puławy	300	10,5
250	(2;5)	Radom-Kędzierzyn-Koźle	250	47,5
160	(2;6)	Radom-Tarnów	200	30,4
95	(1;2)	Warszawa-Radom	0	18,1
95	(2;1)	Radom-Warszawa	0	18,1
60	(9;7)	Świnoujście-Szczecin	900	11,4
14	(10;8)	Medyka-Przemyśl	400	2,7

odległości	Łuk (p;i)	Łuk Trasy	Ilośc składu	K jedn. Składu
450	(7;1)	Szczecin-Warszawa	2	4000,0
500	(7;2)	Szczecin-Radom	7	4500,0
300	(8;1)	Przemyśl-Warszawa	0	2700,0
215	(8;2)	Przemyśl-Radom	4	1940,0

koszt jednostkowy droga	1[km] x 1[T] x 0,19 [zł/km]
koszt jednostkowy kolei	1[km] x 1[T] x 0,09 [zł/km]

Węzeł numer	Węzeł nazwa	LHS	Podaż	Popyt	Możliwości przyjęcia
1	Warszawa	0		0	0
2	Radom	350		300	350
3	Włocławek	200		150	200
4	Puławy	300		200	300
5	Kędzierzyn-Koźle	250		150	300
6	Tarnów	200		200	200
7	Szczecin	0		0	0
8	Przemyśl	0		0	0
9	Świnoujście	900	900		
10	Medyka	400	400		

Koszt przewozów samochodowych	37585,0
Koszt przewozów kolejowych	47260,0
Koszt przewozów ogółem	84845,0

ROZWIĄZANIE | DANE

Fig. 7. Optimal transportation plan for fertilizers
Source: Own elaboration

Figure 8 shows a network of arrows with non-zero optimal flows. The arrows show the flow rates between the nodes by numbering accepted in the task.

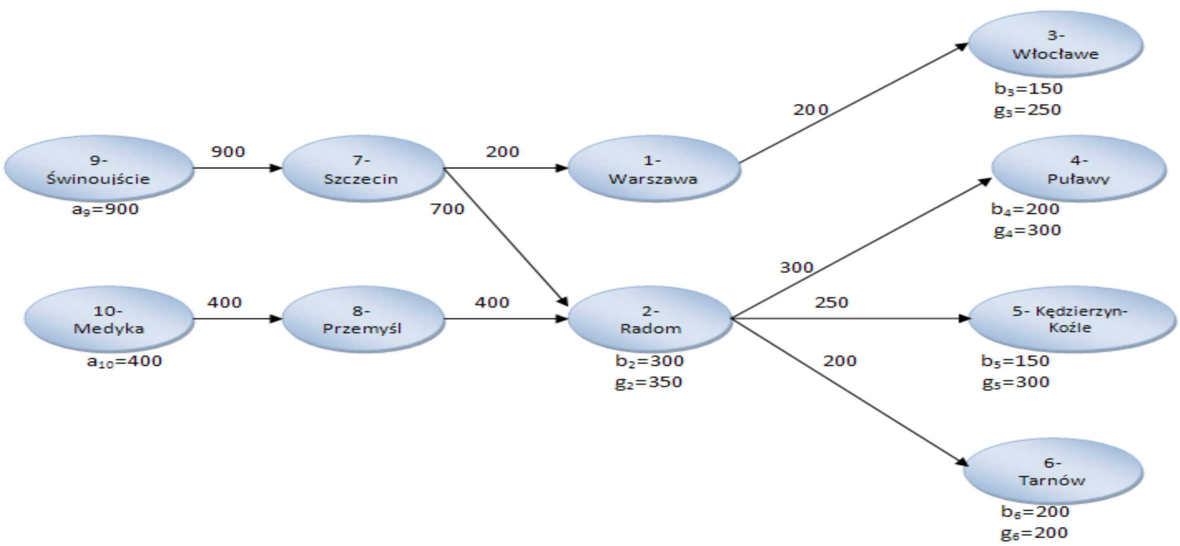

Fig. 8. Network of arrows with non-zero optimal flows and their size
Source: Own elaboration

The Solver tool creates a report of the specified type and places each report in a separate worksheet sheet. We can receive the report of:

- results - creates a list containing the target cell and cells of variables along with their original and final values, restrictions, and constraint information.

- sensitivity - informs about the sensitivity of the solution to slight changes in the formula contained in the target cell field in the Solver Parameters dialog or in the constraints. This report is not generated for models that have total limitations. In the case of linear models report includes reduced costs, apparent prices, objective factor (with an acceptable growth and reduction) and right-hand ranges of constraints.
- limits -creates a list containing the target cell and cells of variables along with their corresponding values, lower and upper limits, and target values. This report is not generated for models that have total limitations. The lower limit is the smallest value that can be accepted by the changing cell having fixed values of the remaining cells of variables and maintained constraints. The upper limit is the highest acceptable value.

Analysis of the positive and negative aspects of the combined transport model. The advantages of the developed model include primarily:

1) The clarity and explicitness of formulated limiting conditions, the function of the target and the sought values - the purpose of the function (cost minimization) proposed by the author, for which the model was built, is a clear set and contains values relatively easy to measure;
2) The popularity of the simplex method used for model optimization, it is a well known method and is commonly used in decision support linear programming. It has an uncomplicated algorithm solution;
3) Use of computer software - a clear advantage of the model; Excel Solver is a popular application among enterprises, which is easy to learn. It enables to build a model quickly and without the knowledge of advanced IT issues.
4) Ease of implementation of the model - the built model of combined transport is relatively easy to implement in the reality, implementation is possible without the need to perform complex operations and processes;
5) Results in the form of synthetic reports - the possibility to easily read the reports of results, sensitivity and constraints;

The proposed optimization model combined transport is not without drawbacks, the most obvious drawbacks include:

1) Formalization of the decision-making process based on the search for the extremum may prove to be not useful. Difficulties are primarily related to the construction of a function that would fully express all the requirements for solving practical problems.

Different qualities that characterize the quality of a decision may not in any way be quantitatively comparable from the point of view of the decision maker.

2) The model does not include random events - the design conception of choice does not take into account the fact that there may be roadblocks in the network that cause delays and increase costs.

The above juxtaposition of advantages and disadvantages of the combined transport system model shows that the number of positive aspects of this concept exceeds the number of its weaknesses. Moreover, some of the disadvantages may not occur throughout the process, so it can be stated that the built model has a good chance of success when used in practice.

SUMMARY

The choice of appropriately selected transport system is an integral part of the logistics processes in the company. Nowadays, due to the increasing trade turnover and the need to keep up with the latest trends in logistics, the importance of transport has increased immensely. The use of the road and rail transport system requires special coordination in transport planning. Achieving the optimal use of the transport system guarantees the company to gain competitive advantage in the market.

Modeling and optimization of transport systems is a time-consuming process, however, it allows to obtain an optimal solution for transport task in the process of distribution of goods.

The model of a combined transport system, designed for this project, based on linear programming for decision support, using the Solver module, demonstrates that it is worthwhile to use the right tools in making this decision, not just base on intuition and experience. The model allows for a complete analysis of the results, through the ability to edit reports.

The developed project, based on theoretical knowledge and analysis of the literature, enriched with the experience of a company struggling with the problem of combining different transport branches and the author's own experience provides some insight into the described process. This view, however, is only a narrow intake of commonly occurring phenomenon, which urges to further discussion and analysis of the problem. Created model has its advantages, but it is not without flaws. I believe that fit the appropriate method for creating the model is difficult, requiring knowledge and experience, hence the proposed solution should be confronted with the results obtained by other methods.

Aim of the paper was completed, the analysis of the problems of modeling and optimization of transport systems was carried out. In addition, by usage of analytical methods a mathematical model of the transport system was built and then optimized basing on the data obtained from the selected company, using the linear optimization method. The enterprise, on the basis of which the model of the combined system has been built and optimized, has the opportunity to compare the results obtained on the basis of their own methods with the proposed method used in the paper and then to use its advantages to improve the transport process. Implementation of the model does not require revolutionary changes in the functioning of the company or significant capital investment and brings potential benefits of the optimization process of combined transport systems.

To sum up, after analyzing the issues of modeling and optimization of transport systems, it is possible to define the directions of logistics transformation related to transport. The changes will be focused on 3 main issues:

1) rational shaping of transport infrastructure development policy,

2) rational shaping of transport policy of carriers,

3) choice of the optimal variant of the transport offer.

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