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MARKOV MODEL OF THE ASSESSMENT OF THE USE OF TRANSPORT MEASURES IN A DISTRIBUTION COMPANY

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Abstract: *In distribution companies, it is particularly important to organize transport processes properly and minimize unnecessary mileage. A high level of readiness of transport means is also required, which guarantees the execution of every order. It enables to achieve rational fleet management, compliance with service intervals and an even workload. The use of transport means is also influenced by the way a company is managed and decisions made at the strategic level. This article shows how the choices made by the management board regarding trade partners (manufacturers) influenced the degree and legitimacy of owned rolling stock use. For this purpose, two different business steps were compared. Markov's processes were used in the study.*
Keywords: *Markov models, exploitation of vehicles, fleet management, vehicles usage*

INTRODUCTION

Properly planned activities are a guarantee of success in all areas of the company's operations. Companies are constantly making decisions that involve many conflicting interests. On the one hand, it is important to minimize costs, and on the other, to improve the quality of both products and services. Maintaining the right balance between these parameters guarantees success in the market and requires, among other things, the selection of the right business partners. The measures of success are the results obtained in relation to the costs incurred. However, they may not fully diagnose existing problems in the company. Therefore, the article presents a method showing how decisions regarding the selection of a producer influenced the purposefulness of using vehicles.

The analysis concerned a company that distributes spare parts for mechanical workshops. It offers a wide range of assortment for cars and vans, as well as motorcycles and scooters, including accessories, liquids and consumables, as well as tools and workshop equipment. The analysis involved one of the company's branches dealing directly with the delivery of orders to car workshops located in the assigned area of responsibility. Distribution is carried out with own vehicles.

After the initial successes, the number of complaints started to grow drastically in the company. The acquired group of customers began to decrease, the dissatisfaction of those who remained rose and profits fell. The reason for this was the strategy adopted by the company. At the beginning, the board decided to cooperate with parts producers/suppliers who offered the lowest prices. The prospect of a long partnership and large orders allowed also to negotiate favorable discounts, which allowed to offer customers very favorable prices. Of course, in the offer catalog, in addition to the cheap assortment, there were also more expensive items, however, it is usually the price criterion that determines the choice of the consumer. That was also the case here. In addition, low prices according to the advertising campaign of the company did not result from their lower quality, but were part of the promotion entering the company's market. In a short time, the company gained a large group of clients, especially among mechanical workshops, to which it primarily dedicated its offer, additionally providing postpone payments, or special loyalty programs that allow collecting points for purchases and allocating them in exchange for a given pool for workshop equipment.

Unfortunately, the initially achieved success turned out to be illusory in a short time, as low-quality parts got damaged and caused many complaints. As a result, not only the company began to lose customers and profits, but also faced the need to conduct numerous complaints. Many of them required complicated procedures with the inclusion of an expert, as the failure of the advertised element occurred usually only after its assembly and caused additional costs related to damage to the parts cooperating in the system, as well as claims for reimbursement of labor costs. Therefore, the management board quickly decided to change the main supplier, choosing a more expensive manufacturer, but guaranteeing a higher quality assortment. Gradually, the company managed to rebuild relations with clients and remain on the market. The tense situation, problems with faulty equipment and customer complaints meant that during the calculation of losses, additional, unplanned operation of transport means was not taken into account. After the changes, the right selection of spare parts manufacturers resulted in a lower number of complaints, but also translated into the degree and method of using vehicles in the company. Mileages that could have been avoided were no longer in place. They contributed not only to the increase of process costs, but also increased the use of transport means, unnecessarily reducing their efficiency and effectiveness.

The analysis compared the activities performed by vehicles in two periods of company's operation. For this purpose Markov models were used both in discrete and continuous physical time. The process evolution in time was also analyzed, calculating the boundary probabilities. The obtained models not only allowed for the description of the analyzed system and the prediction of selected logistic indicators, but also showed the directions of possible improvements.

In addition, the analysis drew attention to the problem affecting many companies which, focused on their main activity, treat accompanying processes as minor or do not analyze them at all, not noticing the hidden potential in them. A review of the literature in this area indicates that this is the case in all types of enterprises, not only private, including state-owned enterprises, and usually concerns merely the assessment of the use of transport means. The research found in the literature concerned, among other things, the evaluation of the operation processes of technical facilities, including, for example, fire engines (Borucka, 2018, p. 397-395), vehicles (Borucka, 2013, p. 39-48) and aircraft (Borucka, 2018, p. 22-30) used in the army. They also concerned elements of logistics in the company (Borucka, 2018, p. 1073-1082; Borucka, 2018, p. 3-19), as well as transport, taking into account factors that determine the flow of liquidity, such as congestion (Mitkow, 2018, p. 501-526), elements related to road safety (Świderski i in., 2018, p. 651-654; Skoczyński i in., 2018, p. 92-97) using the subject (Waśniewski, 2011, p. 223-233), and even waste management (Mikosz, 2008, p. 1-12). Thus, in each system there are areas that are not examined, despite the fact that the obtained results could significantly improve their functioning (Szczepański i in., 2019, p. 1-9).

1. MARKOV MODEL IN DISCRETE TIME

Markov processes are such a group of stochastic processes $\{X_t\}_t$, which fulfill Markov's property, i.e. assumption about the lack of history (Borucka, 2018, p. 3-19; Filipowicz, 1996). For each moment t_0, the probability of change to any system location for $t > t_0$ depends only on its position at time $t = t_0$, and it does not depend on how this process took place in the past. Thus, the random process $\{X(t_0 + \tau), t \epsilon T\}$ is called the Markov process when for any finite sequence of moments $t_1 < t_2 < \cdots < t_n$ $(t_1, t_2 \ldots t_n \epsilon T)$ and any real numbers $(x_1, x_2, \ldots, x_n)$ equality (1) occurs:

$$P[X(t_n) < t_n | X(t_{n-1}) = x_{n-1}, X(t_{n-2}) = x_{n-2}, \ldots, X(t_1) = x_1] = P[X(t_n) = x_n | X(t_{n-1}) = x_{n-1} \quad (1)$$

which means that the conditional probability distribution of a random variable $X(t_n)$ at the moment t_n depends only on the probability distribution of a random variable $X(t_{n-1})$ at time t_{n-1}, and does not depend on probability distributions of a random variable, which process took in moments $t_1, t_2, \ldots, t_{n-2}$.

The construction of the Markov model started with the separation of possible operational states in which the vehicles delivering are staying. Their minimum number was distinguished - 3 disjoint states allowing to analyze the delivery process before and after the changes. This is a model that is sufficient for such analysis. According to the principle of estimation, one should strive for the simplest form of the model, without unnecessarily complicating its structure, if its form meets the needs of the researcher. In connection with the above, the following conditions have been proposed:

S1 - performance of the delivery resulting from the order,
S2 - handling activities,
S3 - complaint trips.

The operational functions of the facility and the system implemented in individual states are as follows:

S1 - concerns the process of delivering the order to the customer of the order. This condition includes driving a car from the company's headquarters to the place where the goods are delivered.

S2 - handling activities that include the loading and unloading of goods, as well as the preparation of necessary documentation, release of goods and settlement of transactions.

S3 - concerns additional, unplanned journeys, which result from complaints reported by customers. Such a process involves taking back the faulty part and compiling full documentation regarding the case. Since the defect often appears only during the use of the vehicle, it is also necessary to take into account all the damage that resulted from it. Everything is also documented by means of photographs, and an appraiser’s presence is often necessary.

According to the company's policy, complaint trips are held as a matter of urgency, at the request of the client or at another date set by him. In the event that all vehicles are on the road, complaints are considered after returning to the base, sometimes as the last route at the end of the working day, also as an additional course. If the complaint is not complicated and does not require additional activities, the basic complaint form is fully sufficient, then the whole procedure does not take much time and can take place during the standard delivery of goods and does not result in an additional course. Most often, however, such situations are accompanied by many emotions, which considerably extends the procedures. Firstly, on the part of the dissatisfied customer whose circumstances forced an unplanned and untimely visit to the workshop, secondly, of course, the mechanics that provided the service. A significant number of complaints in the first period meant that subsequent failures severely strained the patience of customers, despite their initial satisfaction with the prices offered.

The first stage of the process description using Markov models is the construction of a matrix of allowed transitions between states. Uncomplicated and quite general, but fully sufficient to investigate the assumed hypotheses, the model caused that all interstate relationships are possible, as shown in the matrix of transitions (Table 1) and in the form of a graph (Figure 2).

Table 1. Matrix of transitions between states

States	S1	S2	S3
S1	0	1	1
S2	1	0	1
S3	1	1	0

Source: Own collaboration.

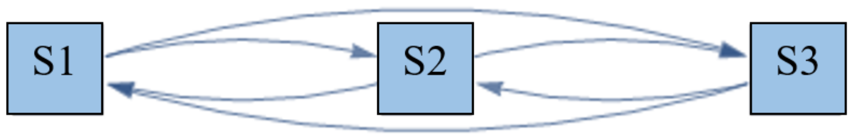

Fig. 1. Graph of transitions between states
Source: Own collaboration.

After defining the possible operational states and existing relations between them, it was necessary to examine the aforementioned Markov property, i.e. lack of memory, meaning that the probability of each event depends only on the previous result, and does not depend on the previous history of the process (Borucka, 2018, p. 1073-1082; Kałuski, 2007). To this end, the Kruskal Wallis test was used to test the independence of a given state from the previous state and from its duration. The obtained result confirmed this independence, therefore in the next stage, the form of distributions of the average duration of the distinguished states was investigated. These distributions proved to be exponential, which authenticated the possibility of using Markov processes. An exemplary histogram, for the duration of state S1, is shown in Fig. 1.

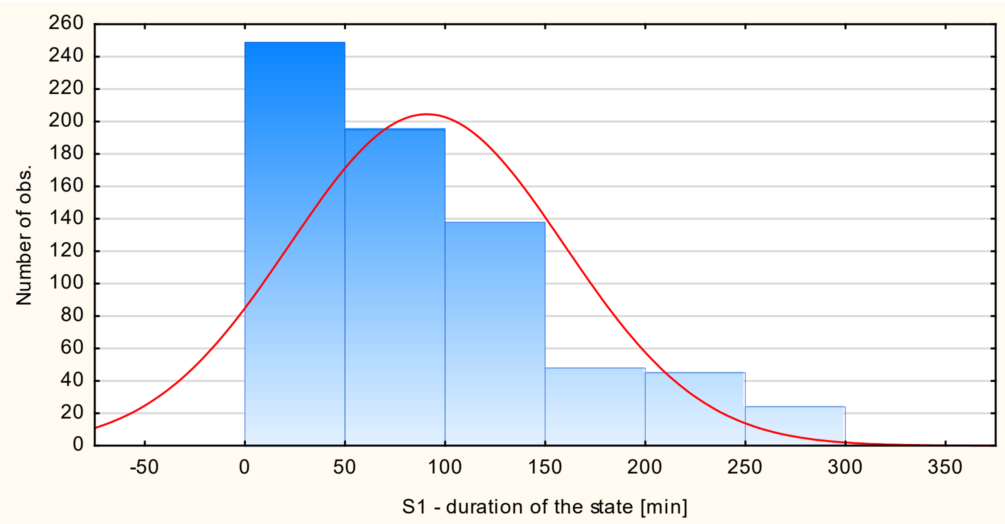

Fig. 2. Distribution of S1 state length period
Source: Own collaboration.

Markov processes with discrete time are called Markov chains. Their mathematical description is the stochastic matrix P whose p_{ij} elements are probabilities of transition from the state $X_n = i$ to the state $X_{n+1} = j$ (Bobrowski, 1985; Borucka, 2018, p. 3-9; Filipowicz, 1996). Therefore, in the next stage of the analysis, the database containing the phase trajectories of objects was converted to the value of the estimators of the pij elements of the probability matrix of the P transitions. The values of these estimators from the sample are the frequencies w_{ij} and go from the

S_i state to the state S_j (Żurek i in., 2017, p. 2343-2352). They are necessary to develop the Markov chain model for the studied process. In this way, a transition probability matrix was obtained for the two analyzed periods of the process of exploitation - before the introduced changes (stage 1) and after their introduction (stage 2). The obtained values of the estimators of both stages are presented in Tab. 3. and Tab. 4, respectively.

Table 2. Values of p_{ij} elements of matrix P, 1st stage of study

p_{ij}	S1	S2	S3
S1	0	0,888	0,112
S2	0,882	0	0,118
S3	0,750	0,250	0

Source: Own collaboration.

Table 3. Values of p_{ij} elements of matrix P, 2nd stage of study

p_{ij}	S1	S2	S3
S1	0	0,936	0,064
S2	0,958	0	0,0418
S3	0,2	0,8	0

Source: Own collabor.ation

Subsequently, two periods were compared with each other. It was examined how the individual estimators of transition probabilities changed, determining the percentage difference between the period before the change was introduced, when the sale was mainly based on the poor quality of the assortment and after it, when the suppliers were changed. The results obtained are presented in Tab. 5.

Table 4. Comparison of two-stage process transition probability values

%	S1	S2	S3
S1		5,38	-42,73
S2	8,69		-64,71
S3	-73,33	220	

Source: Own collaboration.

The values obtained refer to the frequency of interstate changes occurring in the system before and after the modification. Above all, it is worth paying attention to the decisive drop in the relation between the state S1 (realization of the sentence) and the state S3 - handling complaints, amounting to as much as 43%. The transitions to state S3 from the state S2, meaning handling activities by as much as 65%, were even more diminished. This means that the vehicles performed much more tasks related to the company's operations in the second stage, only sporadically performing complaint transport. The number of entries from the state S3 to the state S1 decreased, which means that more complaints were dealt with at a distance without the necessity to come to the place. Hence, the increase of transitions from S3 to S2.

The Markov chain models estimated for both analyzed stages proved to be ergodic, which enabled a long-term forecast of the evolution of the studied system (Grabski i in., 2009). For this purpose, the probability limits pj allowing for prediction of the implementation of the process in the long-term were calculated. The calculations were made using the Microsoft Excel Solver add-in and confirmed in the Mathematica program which allows performing complex calculations in a mathematical environment. The results obtained were compared with each other as shown in Tab. 6 and Fig. 2.

Table 5. Ergodic probabilities of p_j Markov chain for 2 stages of the process

p_j		S1	S2	S3
before	p_j	0,4613	0,4355	0,1032
	$p_{j\%}$	46,13	43,55	10,32
after	p_j	0,4699	0,4799	0,0501
	$p_{j\%}$	46,99	47,99	5,01
modification		1,86	10,20	-51,39

Source: Own collaboration.

According to the Markov model in discrete time, the system aims primarily at staying in two states, in the state of S1 (delivery) and in the state of S2 (including handling activities) at a very similar level. The system's boundary probabilities after changes are higher for these states in relation to the values before changes by 2% and 10% respectively. The largest difference concerns the state of S3 (complaints) and is over 50%. This means that the frequency of complaints at the borderline system will be half as much, as shown in Figure 3. Long-term forecasts for the Markov chain are therefore satisfactory from the entrepreneur's point of view.

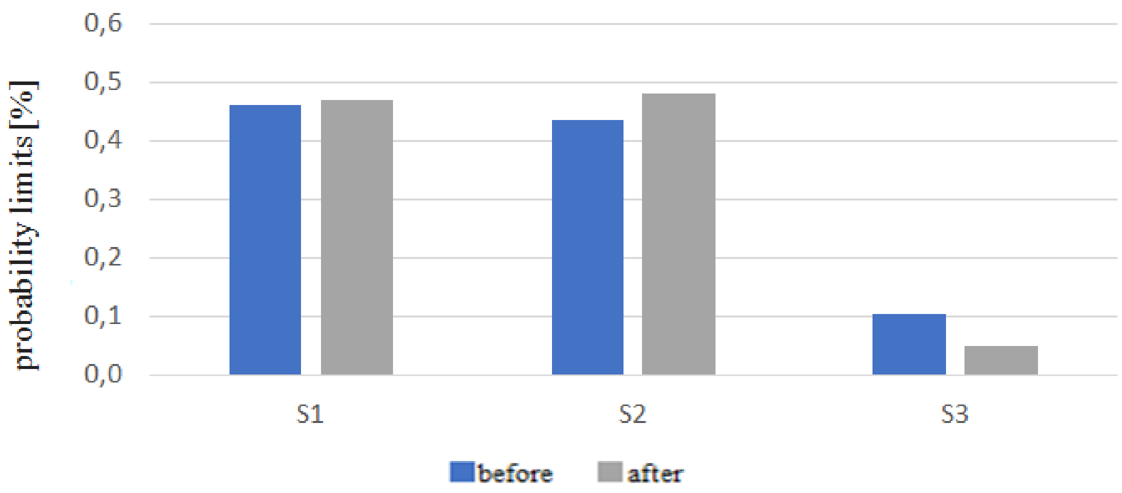

Fig. 3. Ergodic probabilities of p_j for 2 stages of the process

Source: Own collaboration.

Average execution times for individual activities were also compared. The results obtained are presented in Table 7. They are again satisfactory and indicate a reduction in average times of manipulation (by 11%) and returns (by 23%), therefore, states that do not translate into profit and even generate costs for the enterprise. However, the average duration of the most important state related to the delivery of the purchased assortment from the point of view of the business increased by 3%. This means an increase in transport and thus an increase in the number of transactions, so also the company's revenues.

Table 6. Comparison of average time length of states S1-S3

State	Mean [min]		Mean [%]
	before	after	modification
S1	87	83	-5,22
S2	16	15	-4,43
S3	24	16	-35,53

Source: Own collaboration.

2. MARKOV MODEL IN CONTINOUS PHYSICAL TIME

The characteristic analysis of the phenomena of the Markov chain only in selected moments of time is often insufficient to describe it completely (Żółtowski i in., 2002). Therefore, the analysis was continued using the Markov model in a continuous physical time. The parameter characterizing such a model, in addition to the transition probability matrix, is also the intensity matrix of transitions Λ with elements of λ_{ij}, which is a function that characterizes the rate of changes in the probabilities of passages $\mathrm{pij(t)}$ (Borucka, 2018, p. 3-19; Filipowicz, 1996). For the process before changes in the values of matrix elements Λ are presented in tab. 8, while after changes in tab. 9.

Table 7. Values of elements λ_{ij} of matrix Λ in 1st stage of the study

p_{ij}	S1	S2	S3
S1	$-0{,}0233$	$0{,}0114$	$0{,}0119$
S2	$0{,}0623$	$-0{,}1217$	$0{,}0594$
S3	$0{,}043$	$0{,}0362$	$-0{,}0792$

Source: Own collaboration.

Table 8. Values of elements λ_{ij} of matrix Λ in 2nd stage of the study

p_{ij}	S1	S2	S3
S1	$-0{,}0235$	$0{,}0122$	$0{,}0113$
S2	$0{,}0643$	$-0{,}1465$	$0{,}0822$
S3	$0{,}0663$	$0{,}0631$	$-0{,}1294$

Source: Own collaboration.

The estimated intensity values made it possible to calculate the ergodic probabilities for the two stages of the process. The obtained results are presented in Tab. 10.

Table 9. Ergodic probabilities p_j for 2 stages of the process in continuous physical time

pj		S1	S2	S3
before	p_j	0,676	0,13	0,194
	$p_{j\%}$	68,4	12,2	19,4
after	p_j	0,736	0,122	0,142
	$p_{j\%}$	73,7	12,1	14,2
modification		7,75	-0,82	-26,80

Source: Own collaboration.

For both stages of the study, the obtained results show the pursuit of the tested system to be primarily in the delivery state (S1), but for the process after changes, this value is 8% higher. The marginal result of handling activities (S2) decreased slightly, which means more efficient procedures. A very favorable change occurred for the complaint status. The desire to stay in this state has decreased by as much as 30%. The obtained results confirm the legitimacy of the introduced changes and reflect their positive impact on owned vehicles, which after the modification of the strategy are used, according to the assumption, primarily to deliver orders to clients.

The final stage of the study was the visualization of the obtained results in the form of determining the characteristic times of aiming the object to the stationary state after a given set of initial states. For this purpose, it is necessary to solve the system of Chapman-Kolmogorow-Smoluchowski equations, which for the studied process have the following matrix form (2):

$$\begin{bmatrix} p_1(t) \\ p_2(t) \\ p_3(t) \end{bmatrix} \cdot \begin{bmatrix} -\lambda_{11} & \lambda_{12} & \lambda_{13} \\ \lambda_{21} & -\lambda_{22} & \lambda_{23} \\ \lambda_{31} & \lambda_{32} & -\lambda_{33} \end{bmatrix} = \begin{bmatrix} p_1'(t) \\ p_2'(t) \\ p_3'(t) \end{bmatrix} \qquad (2)$$

The correct analytical solution of the CH-K-S system of equations with restriction, the condition for normalization was determined using the Mathematica Markov continuous module. For the test, it was assumed that at the initial moment t = 0 the process X (t) is in state S1. This allowed to plot a function that shows the pursuit process for a limit state, respectively for the state S1 (Figure 4) of state S2 (Figure 5) and status S3 (Figure 6).

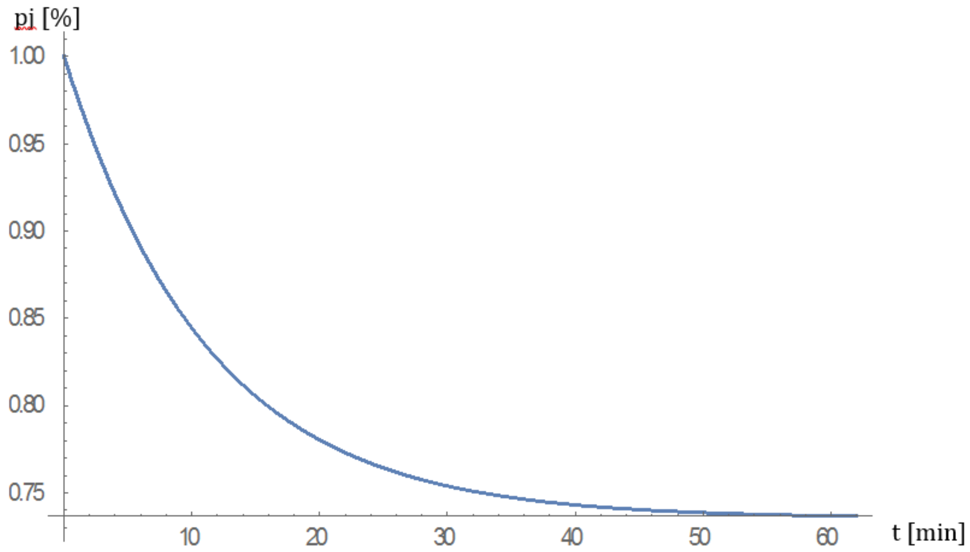

Fig. 4. Evolution of probability of the system in state S1
Source: Own collaboration.

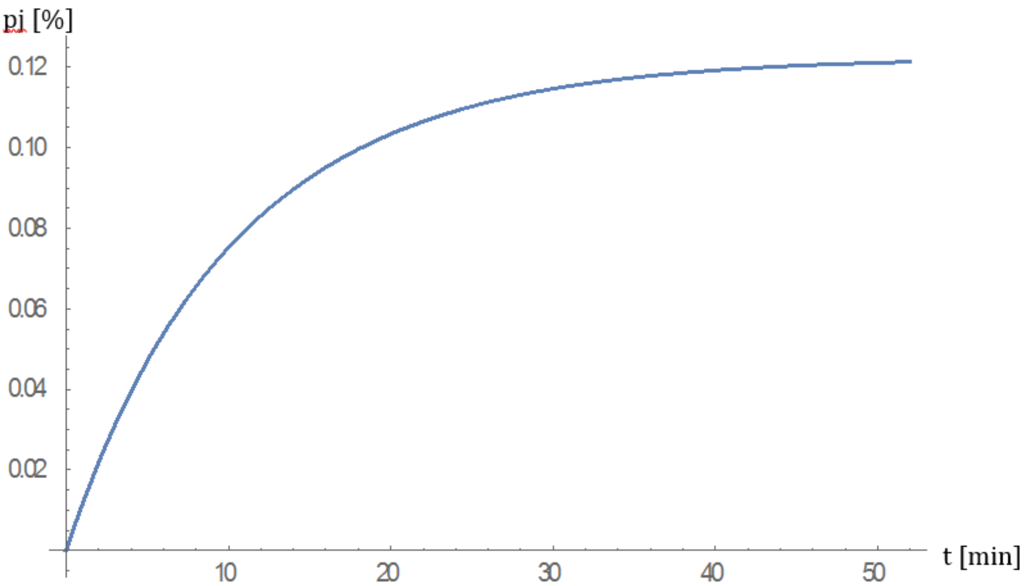

Fig. 5. Evolution of probability of the system in state S2
Source: Own collaboration.

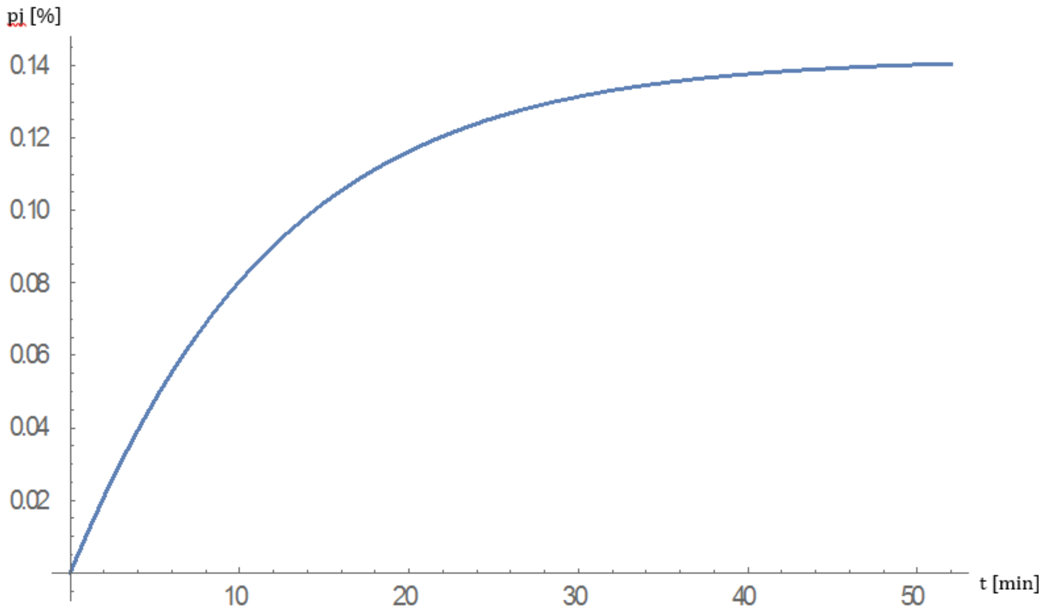

Fig. 6. Evolution of probability of the system in state S3
Source: Own collaboration.

The probability of the S1 state (delivery) decreases to the limit value in about 60 minutes. Asymptotic values for other states are set at a similar time. Thus, after an hour of enforcing the initial state S1, the system reaches equilibrium.

CONCLUSION

The research, in the first place, showed how to use modeling using Markov processes to describe selected elements of the distribution system. The focus was primarily on transporting vehicles and the method of their use, depending on the strategy adopted in the company regarding the selection of the manufacturer's spare parts. Decisions in this area had strongly affected the overall functioning of the company, including the way vehicles were used.

Low-quality spare parts resulted in failures causing numerous complaints. As a result, vehicles, instead of order deliveries, were used to handle complaints, which generated a lot of additional trips, resulted in off-road routes and contributed to increased consumption of transport means, and as a result caused additional costs for the company generated by more frequent service repairs of braking, steering systems and drive, as well as the exchange of filters and operating fluids.

After the changes, the time that was previously devoted to handling the complaint was largely intended for subsequent deliveries. It had become an activity generating costs that bring profits, and the use of vehicles had become a normal part of the logistics process and was fully justified.

Calculated forecasts according to the Markov model for discrete and continuous time are similar. They generate the same forecast of the most probable states, but with different values of limit probabilities. According to the forecast for the Markov chain (for the abstract discrete time of subsequent state changes) and according to the Markov model in a continuous time, the system after changes is aimed at being present mainly in the state S1 (delivery) and S2 (handling activities). For the value chain, these probabilities are close to each other and amount to 47 and 48%, while for the process, which in addition to the transitions frequency also includes average durations of operating states, these values are 74 and 12%.

The most important change, however, concerns the state of S3, for which the limit value for the Markov chain decreased by over 50% and for the continuous time by 27%. Thus, not only the frequency of being in this state changed, but also its intensity.

The analysis showed that even a simple Markov model creates the opportunity to describe the system of functioning of transport means, provides reliable information on their use, and can be the basis for determining the main directions of improvements, effectively indicating areas that increase the efficiency of their use.

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